

# Magneto-transport in a quantum network: Evidence of a mesoscopic switch

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We investigate magneto-transport properties of a three-arm mesoscopic ring where the upper and lower sub-rings are threaded by Aharonov-Bohm fluxes  $\phi_1$  and  $\phi_2$ , respectively, within a non-interacting electron picture. A discrete lattice model is used to describe the quantum network in which two outer arms are subjected to binary alloy lattices and the middle arm contains identical lattice sites. It is observed that the presence of the middle arm provides localized states within the band of extended regions and they lead to the possibility of switching action from the high conducting state to a low one and vice versa. This behavior is justified by studying persistent current in the network. Both the total current and individual currents in three separate branches are computed by using second-quantized approach and our idea can be utilized to study magnetic response in any complicated quantum network. The nature of localized eigenstates are also investigated in terms of probability amplitude in different sites of the geometry.

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## I. INTRODUCTION

Theoretical and experimental investigations in low-dimensional systems lead to the opportunity of scrutinizing various novel quantum mechanical effects [1, 2] in a tunable way. Persistent current being one such quantum mechanical phenomenon observed in normal metal single-channel rings and multi-channel cylinders in the presence of Aharonov-Bohm (AB) flux  $\phi$ . Prior to the experimen-

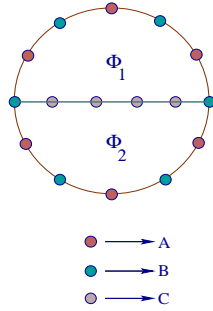


FIG. 1: (Color online). Schematic view of a quantum network where the upper and lower sub-rings are threaded by AB fluxes  $\phi_1$  and  $\phi_2$ , respectively. Both the upper and lower arms are subjected to binary alloy lattices, while the middle arm contains identical lattice sites. The filled colored circles correspond to the positions of the atomic sites.

tal verification, first theoretical evidence regarding the appearance of a non-decaying current in normal metal rings has been illustrated by Büttiker, Imry and Landauer [3], and, in the sub-sequent years many other theoretical works have been carried out by several groups [4–16] to understand the actual mechanism behind it. The

experimental justification for the phenomenon of non-decaying current in metallic rings/cylinders has been established quite in late. It has been first examined by Levy *et al.* [17] and later many other experiments [18–20] have been performed to justify the existence of non-dissipative current in such systems.

Although the studies involving the mesoscopic rings have already generated a wealth of literature there is still need to look deeper into the problem to address several important issues those have not been explored earlier. It is well known that in presence of random site-diagonal disorder in an one-dimensional mesoscopic ring all the energy levels are localized [21], and accordingly, the persistent current amplitude gets reduced enormously compared to that of an ordered ring. But there are some class of materials which can exhibit extended eigenstates along with the localized energy levels, and these materials may provide several interesting issues, mainly to provide a localization to delocalization transition and vice versa. For example, in a pioneering work Dunlap *et al.* [22] have shown that in an one-dimensional tight-binding model with two uncorrelated random site energies and a constant nearest-neighbor hopping integral, an initially localized particle can become delocalized depending on the parameter values. They have proposed that any physical system which can be described by the random dimer model should exhibit the transmission resonances and a huge enhancement in the transmission takes place when the Fermi level coincides with the unscattered states. In a consecutive year Wu *et al.* [23] have argued that the random dimer model can be used to explain the insulator to metal transition in polyaniline as a result of the movement of the Fermi level to extended region. Later, several other works [24–26] have also been carried out in such type of materials to exhibit many important physical results.

In the present work we consider a three-arm mesoscopic ring in which two outer arms are subjected to

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binary alloy lattices and the middle one contains identical lattice sites, and, we show that due to the presence of the middle arm quasi-localized energy eigenstates are observed within the band of extended regions. It leads to the possibility of getting a switching action from the high conducting state to a low one and vice versa as a result of the movement of the Fermi level. We illustrate this behavior by studying persistent current in the quantum network and explore the nature of energy eigenstates in terms of the probability amplitude in different lattice sites of the geometry. Our present analysis can be utilized to study magnetic response in any complicated quantum network.

In what follows we describe the procedure and the results. In Section II, we present the model and the method, which include the Hamiltonian and the formulation of persistent current in individual branches of the network. In Section III we discuss the results, and in Section IV we draw our conclusions.

## II. MODEL AND THEORETICAL FORMULATION

### A. The model and the Hamiltonian

Let us refer to Fig. 1. A three-arm mesoscopic ring where the upper and lower sub-rings are threaded by AB fluxes  $\phi_1$  and  $\phi_2$ , respectively. The outer arms are subjected to binary alloy lattices (lattices with A and B types of atoms) and the middle arm contains identical lattice sites (atomic sites labeled by C). The filled colored circles correspond to the positions of the atomic sites. Within a tight-binding framework the Hamiltonian for such a network reads as,

$$H = \sum_j \epsilon_j c_j^\dagger c_j + t \sum_j \left( c_j^\dagger c_{j+1} e^{-i\theta_1} + h.c. \right) + \sum_l \epsilon_l c_l^\dagger c_l + v \sum_l \left( c_l^\dagger c_{l+1} e^{-i\theta_2} + h.c. \right) \quad (1)$$

where,  $\epsilon_j$  represents the site energy for the outer arms, while for the middle arm it is assigned by  $\epsilon_l$ . We call  $\epsilon_j = \epsilon_A$  for the atomic sites labeled by A, while it will be  $\epsilon_B$  for the B type of atoms. On the other hand,  $\epsilon_l = \epsilon_C$  for the atomic sites those are referred by C atoms.  $t$  and  $v$  are the nearest-neighbor hopping integrals in the outer and middle arms, respectively. Due to the presence of magnetic fluxes in two sub-rings, phase factors  $\theta_1$  and  $\theta_2$  are appeared into the Hamiltonian. They are expressed as follows:  $\theta_1 = 2\pi(\phi_1 + \phi_2)/(N_U + N_L)\phi_0$  and  $\theta_2 = 2\pi(\phi_1 - \phi_2)/2N_M\phi_0$ . Here the fluxes are measured in terms of the elementary flux-quantum  $\phi_0 (= ch/e)$ , and,  $N_U$ ,  $N_M$  and  $N_L$  represent the total number of single bonds (each single bond is formed by connecting two neighboring lattice sites) in the upper, middle and lower arms, respectively. It reveals  $N_U + N_M + N_L - 1$  number of atomic sites in the quantum network.  $c_j^\dagger$  ( $c_j$ ) corresponds to the creation (annihilation) operator of an elec-

tron at the site  $j$ , and, a similar definition also goes for the atomic sites  $l$ .

### B. Calculation of persistent current

In the second quantized notation the general expression of charge current operator is in the form [27],

$$I = \frac{2\pi i e \alpha}{L} \sum_n \left( c_n^\dagger c_{n+1} - c_{n+1}^\dagger c_n \right). \quad (2)$$

Here,  $L$  is the length of the arm in which we are interested to calculate the current and  $\alpha$  represents the nearest-neighbor hopping strength. It ( $\alpha$ ) is equal to  $t$  for the outer arms, while for the middle arm it becomes identical to  $v$ . Therefore, for a particular eigenstate  $|\psi_k\rangle$  the persistent current becomes,  $I^k = \langle \psi_k | I | \psi_k \rangle$ , where  $|\psi_k\rangle = \sum_p a_p^k |p\rangle$ . Here  $|p\rangle$ 's are the Wannier states and  $a_p^k$ 's are the corresponding coefficients.

Following the above relations, now we can write the expressions for persistent current in the individual branches for a particular eigenstate  $|\psi_k\rangle$ . They are as follows.

For the upper arm:

$$I_U^k = \frac{2\pi i e t}{N_U + N_L} \sum_j \left( a_j^{k*} a_{j+1}^k e^{-i\theta_1} - h.c. \right) \quad (3)$$

where, summation over  $j$  runs from 1 to  $N_U$ .

In the case of middle-arm:

$$I_M^k = \frac{\pi i e v}{N_M} \sum_l \left( a_l^{k*} a_{l+1}^k e^{-i\theta_2} - h.c. \right) \quad (4)$$

here, the net contribution comes from  $N_M$  bonds.

Finally, for the case of lower arm:

$$I_L^k = \frac{2\pi i e t}{N_U + N_L} \sum_j \left( a_j^{k*} a_{j+1}^k e^{-i\theta_1} - h.c. \right) \quad (5)$$

In this case the net contribution comes from the lower bonds. The lattice constant  $a$  is set equal to 1.

At absolute zero temperature ( $T = 0$  K), the net persistent current in any branch of the quantum network for a particular electron filling can be obtained by taking the sum of individual contributions from the energy levels with energies less than or equal to Fermi energy  $E_F$ . Therefore, for  $N_e$  electron system total persistent in any branch becomes  $I_\beta = \sum_k^{N_e} I_\beta^k$ , where  $\beta = U, M$  and  $L$ , for the upper, middle and lower arms, respectively. Once  $I_U$ ,  $I_M$  and  $I_L$  are found out, the net persistent current for the full network can be easily obtained simply by taking the sum of these three terms i.e., the total current  $I_T = I_U + I_M + I_L$ .

The net persistent current ( $I_T$ ) can also be determined in some other ways as available in the literature. Most

probably the easiest way of calculating persistent current is the case where first order derivative of ground state energy with respect to magnetic flux is taken into account [7, 8]. But in our present scheme, the so-called second-quantized approach, there are some advantages compared to other available procedures. Firstly, we can easily calculate persistent current in any branch of a network. Secondly, the determination of individual responses in separate arms helps us to elucidate the actual mechanism of electron transport in a transparent way.

In the present work we investigate all the essential feature of magneto-transport at absolute zero temperature and choose the units where  $c = e = h = 1$ . Throughout the numerical work we set  $t = v = -1$  and measure the energy scale in unit of  $t$ .

### III. NUMERICAL RESULTS AND DISCUSSION

#### A. Quantum network with $\epsilon_A = \epsilon_B = 0$

We start with the model quantum system where  $\epsilon_A$  and  $\epsilon_B$  are identical to each other. Here we set  $\epsilon_A = \epsilon_B = 0$ . To have a clear idea of such a model quantum system,

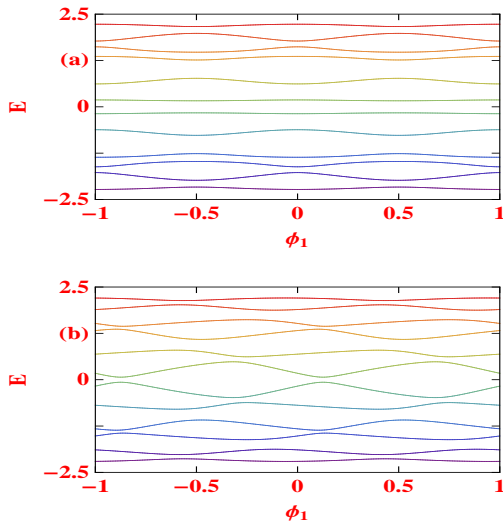


FIG. 2: (Color online). Energy levels as a function of flux  $\phi_1$  for a three-arm ring with  $\epsilon_A = \epsilon_B = 0$  considering  $N_U + N_L = 10$  and  $N_M = 3$ , where (a) and (b) correspond to  $\phi_2 = 0$  and  $\phi_0/4$ , respectively.

first we illustrate the behavior of energy spectra as a function of flux  $\phi_1$  for different values of flux  $\phi_2$  threaded by the lower sub-ring. The results are presented in Fig. 2, where (a) and (b) correspond to  $\phi_2 = 0$  and  $\phi_0/4$ , respectively. In the absence of flux  $\phi_2$ , energy levels near the edges of the spectrum become more dispersive than the levels lie in the central region (see Fig. 2(a)) and they are almost non-dispersive with respect to flux  $\phi_1$ . This feature emphasizes that the persistent current amplitude becomes highly sensitive to the electron filling i.e., the

Fermi energy  $E_F$  of the system, since the current is directly proportional to the slope of the energy levels [7]. The situation becomes much more interesting when we add a magnetic flux in the lower sub-ring. Here, the energy levels near the central region of the spectrum becomes more dispersive in nature than the energy levels

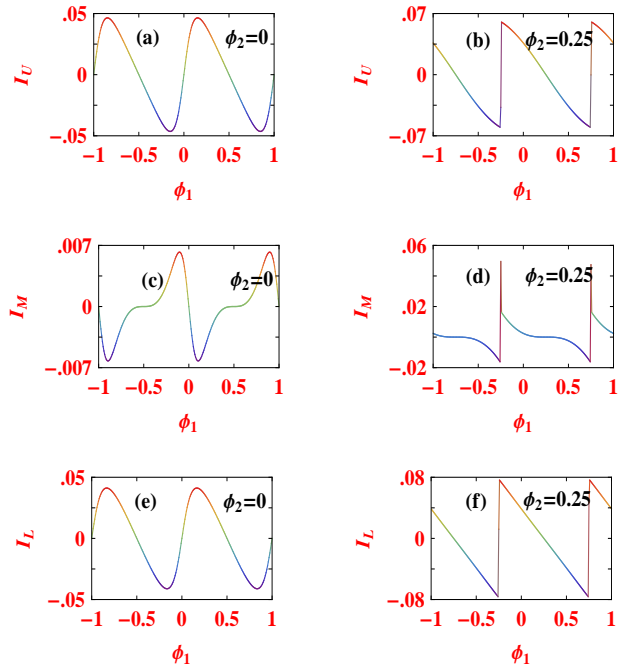


FIG. 3: (Color online). Persistent current in different arms as a function of  $\phi_1$  for a three-arm ring with  $\epsilon_A = \epsilon_B = 0$  in the half-filled band case considering  $N_U + N_L = 60$  and  $N_M = 25$ .

near the edges (Fig. 2(b)), and it increases gradually with flux  $\phi_2$ , which gives a possibility of getting higher current amplitude with increasing the total number of electrons  $N_e$  in the system. A similar kind of energy spectrum is also observed if we plot the energy levels as a function of flux  $\phi_2$  instead of  $\phi_1$ , keeping  $\phi_1$  as a constant. All these energy levels exhibit  $\phi_0 (= 1, \text{ in our choice of units } c = e = h = 1)$  flux-quantum periodicity. Thus, for such a simple quantum network persistent current amplitude might be regulated for a particular filling simply by tuning the magnetic flux threaded by anyone of two such sub-rings, and, its detailed descriptions are available in the sub-sequent parts.

In Fig. 3 we present the variation of persistent current in individual arms of the three-arm quantum network as a function of flux  $\phi_1$  for some fixed values of  $\phi_2$ . The panels from the top correspond to the results for the upper, middle and lower arms, respectively, and in all these cases the current is determined for the half-filled band case i.e.,  $N_e = 42$ . The left column represents the current for  $\phi_2 = 0$ , and the right column gives the current when  $\phi_2$  is fixed at  $\phi_0/4$ . From the spectra we notice that in some cases current shows continuous like behavior while in some other cases it exhibits saw-tooth like nature as a

function of flux  $\phi_1$  threaded by the upper sub-ring. This saw-tooth or continuous like feature solely depends on the behavior of the ground state energy for a particular filling ( $N_e$ ). It is to be noted that in a conventional ordered AB ring we always get saw-tooth like behavior of persistent current irrespective of the filling of the electrons [7]. In

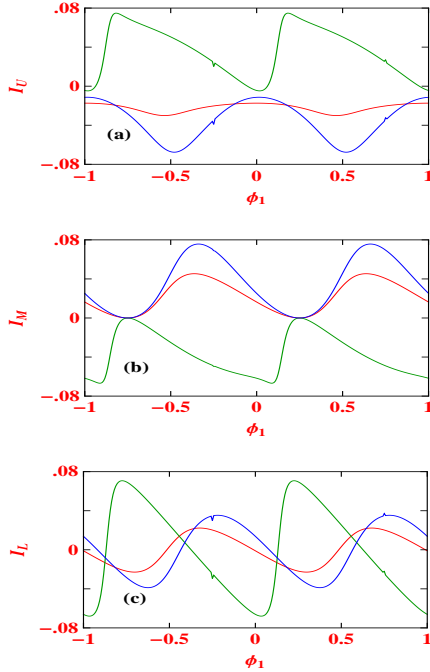


FIG. 4: (Color online). Persistent current in different arms as a function of  $\phi_1$  for a three-arm ring with  $\epsilon_A = \epsilon_B = 0$  considering  $N_U + N_L = 40$  and  $N_M = 17$ . The red, green and blue curves correspond to  $N_e = 10, 15$  and  $20$ , respectively. For all these spectra  $\phi_2$  is set at  $\phi_0/4$ .

the saw-tooth variation a sudden change in direction of persistent current takes place across a particular value of magnetic flux which corresponds to a phase reversal from the diamagnetic nature to the paramagnetic one or vice versa. In our three-arm geometry we also observe that though the current in the upper arm or in the lower arm is not so sensitive to the flux  $\phi_2$ , but the current amplitude in the middle arm changes remarkably, even an order of magnitude, in presence of flux  $\phi_2$ , which leads to a net larger current since the total current is obtained by adding the contributions from the individual arms.

To explore the filling dependent behavior of persistent current, in Fig. 4 we display persistent current in three different arms as a function of flux  $\phi_1$  for a typical value of  $\phi_2$ . Here,  $\phi_2$  is set at  $\phi_0/4$ . The red, green and blue lines represent the currents for  $N_e = 10, 15$  and  $20$ , respectively. The current in different arms shows quite a complex structure which strongly depends on the electron filling as well as magnetic flux  $\phi_2$ . In all these cases persistent current provides  $\phi_0$  flux-quantum periodicity, like a traditional single-channel mesoscopic ring or a multi-channel cylinder.

## B. Quantum network with $\epsilon_A \neq \epsilon_B$

Now we focus our attention to the geometry where site energies in the outer arms are no longer identical to each other i.e.,  $\epsilon_A \neq \epsilon_B$ . In this case energy spectrum gets modified significantly compared to the previous one where site energies are uniform ( $\epsilon_A = \epsilon_B$ ). To illustrate it in Fig. 5 we plot the energy-flux characteristics for a three-arm quantum network considering  $\epsilon_A = -\epsilon_B = 1$ , where (a) and (b) correspond to the identical meaning as given in Fig. 2. Since the upper and lower arms of the network are subjected to the binary alloy lattices we get two sets of discrete energy levels spaced by a finite gap around  $E = 0$  (see Fig. 5). Quite interestingly we see that

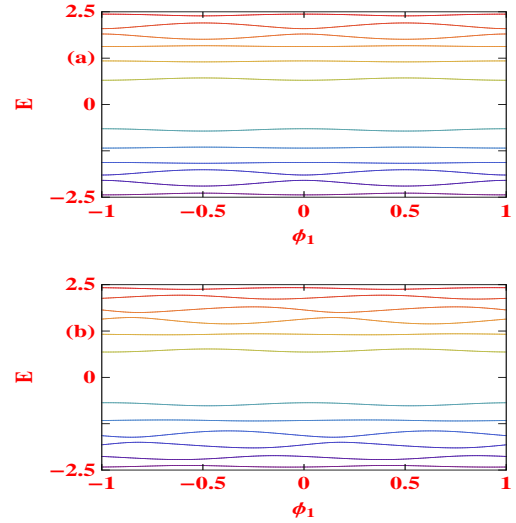


FIG. 5: (Color online). Energy levels as a function of flux  $\phi_1$  for a three-arm ring with  $\epsilon_A = -\epsilon_B = 1$  considering  $N_U + N_L = 10$  and  $N_M = 3$ , where (a) and (b) correspond to  $\phi_2 = 0$  and  $\phi_0/4$ , respectively.

the energy levels near the two extreme edges of the spectrum become more dispersive in nature than the energy levels situated along the inner region. With increasing the difference in site energies ( $|\epsilon_A - \epsilon_B|$ ), we get more less dispersive energy levels in the inner region and for large enough value of  $|\epsilon_A - \epsilon_B|$  these levels become almost non-dispersive and they practically contribute nothing to the current. Thus, for such a system a mixture of quasi-extended and quasi-localized energy levels are found out and it can provide a very large or almost zero current depending on the electron filling. For a very large system size, the energy separation between two successive levels in each set of discrete energy levels gets reduced and we get two quasi-band of energies separated by a finite gap, where the gap is controlled by the parameter values. It is important to note that, unlike the previous one (Fig. 2), for this case the energy spectrum is not so sensitive to flux  $\phi_2$  (Fig. 5). The presence of C-type of atoms in the middle arm which divides the binary alloy ring into two sub-rings is responsible for the existence of

quasi-localized energy levels near the inside edges of two quasi-band of energies. Thus we get more non-dispersive energy levels with increasing the length of the middle arm.

The existence of nearly extended and localized states becomes much more clearly visible from our current-flux

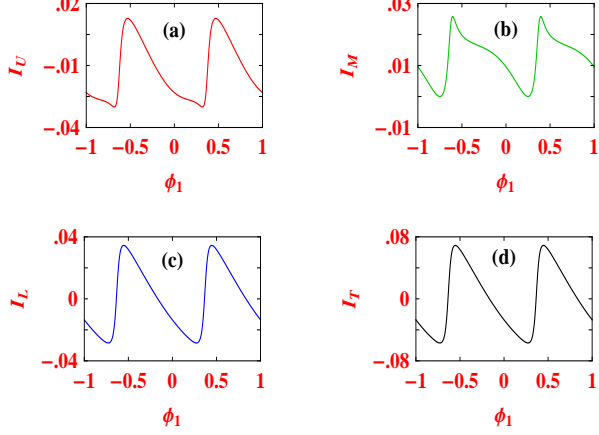


FIG. 6: (Color online). Current-flux characteristics of a three-arm ring with  $\epsilon_A = -\epsilon_B = 1$  considering  $N_U + N_L = 60$ ,  $N_M = 25$  and  $\phi_2 = \phi_0/4$  in the quarter-filled ( $N_e = 21$ ) band case, where (a)-(d) correspond to the currents in the upper, lower and middle arms and in the full system, respectively.

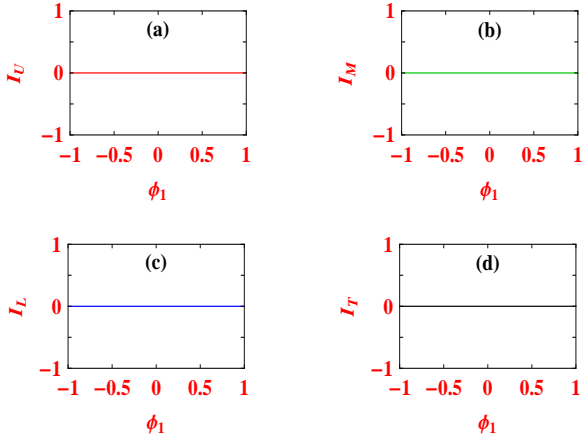


FIG. 7: (Color online). Current-flux characteristics of a three-arm ring with  $\epsilon_A = -\epsilon_B = 1$  in the half-filled ( $N_e = 42$ ) band case for the same parameter values used in Fig. 6.

spectra. As illustrative example, in Fig. 6 we display the variation of persistent current in individual arms including the total current of a three-arm ring considering  $\epsilon_A = -\epsilon_B = 1$  for the quarter-filled ( $N_e = 21$ ) band case. The flux  $\phi_2$  is set equal to  $\phi_0/4$ . From the spectra it is clearly observed that the current in each arm provides a non-zero value (Figs. 6(a)-(c)), and accordingly, the system supports a finite current as shown in Fig. 6(d). The situation becomes completely opposite

when the filling factor is changed. Quite remarkably we notice that persistent current almost vanishes in three separate branches which provides almost vanishing net current in the half-filled band case. The results are illustrated in Fig. 7, where (a)-(d) correspond to the identical meaning as in Fig. 6. The vanishing nature at half-filling

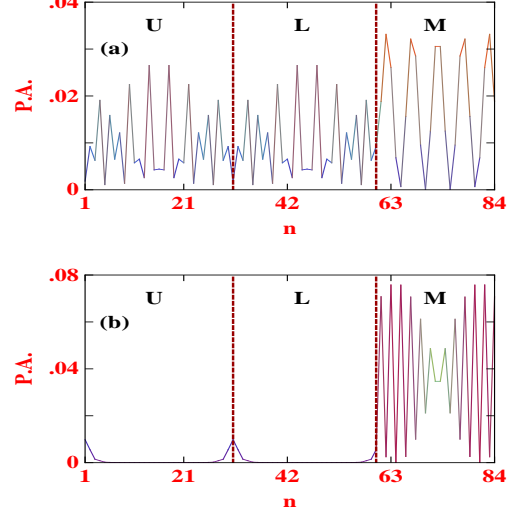


FIG. 8: (Color online). Probability amplitude (P.A.) as a function of site index ( $n$ ) for a three-arm ring with  $\epsilon_A = -\epsilon_B = 1$  considering  $N_U + N_L = 60$  and  $N_M = 25$  when  $\phi_1$  and  $\phi_2$  are set at  $\phi_0/4$ . (a) and (b) correspond to the results of 21-st and 42-nd eigenstates, respectively.

and the non-vanishing behavior of current when the system is quarterly filled can be easily understood from the following argument. The total current in any branch or in the complete system mainly depends on the contributions coming from the higher occupied energy levels, while the contributions from the other occupied energy levels cancel with each other. Therefore, for the quarter-filled band case, the net contribution comes from the energy levels which are quasi-extended in nature and a non-zero current appears. On the other hand, for the half-filled band case, the net contribution arises from the levels those are almost localized, and hence, nearly vanishing current is obtained. Thus, we can emphasize that the three-arm ring leads to a possibility of getting high-amplitude to low-amplitude (almost zero) persistent current simply by tuning the filling factor  $N_e$  i.e., the Fermi energy  $E_F$ , and hence the network can be used as a mesoscopic switch.

To ensure the extended or localized nature of energy eigenstates, finally we demonstrate the variation of probability amplitude (P.A.) of the eigenstates as a function of site index  $n$ . The probability amplitude at any site  $n$  for a particular eigenstate  $|\psi_k\rangle$  is obtained from the factor  $|a_n^k|^2$ . Here we analyze the localization behavior for two different energy eigenstates, viz, 21-st and 42-nd states. For the first one the energy is located well inside a quasi-band, while for the other the energy is placed at the edge of this band. The results are given in Fig. 8 for



a three-arm ring with 84 atomic sites. The red dashed lines are used to identify three distinct regions of the network. In Fig. 8(a) we see that the probability amplitude becomes finite for any site  $n$  which indicates that the energy eigenstate is quasi-extended. While, for the other state (42-nd) the probability amplitude almost vanishes for the upper and lower arms of the network. Only in the atomic sites of the middle arm we get finite probability. Therefore, this state will not contribute anything to the current and we can refer the state as a localized one.

#### IV. CONCLUSION

To summarize, in the present work we have explored the magneto-transport properties of a three-arm quantum ring in the non-interacting electron framework. The upper and lower sub-rings of the network are threaded by magnetic fluxes  $\phi_1$  and  $\phi_2$ , respectively. We have used a single-band tight-binding Hamiltonian to illustrate the

model quantum system, where the outer arms are subjected to the binary alloy lattices and the middle arm has identical lattice sites. In the absence of the middle arm, all the energy eigenstates are extended, but the inclusion of the middle arm provides localized states within the band of extended states and they provide a possibility of getting high conducting state to the low one upon the movement of the Fermi energy. Thus, the system can be used as a mesoscopic switch. Here, we have verified the switching action from high- to low-conducting state and vice versa by investigating the persistent current in the network for different band filling. We have numerically computed both the total current and the individual currents in separate branches by using second-quantized approach. We hope our present analysis may be helpful for studying magneto-transport in any complicated quantum network. Finally, we have also examined the nature of the energy eigenstates in terms of the probability amplitude in different sites of the geometry.

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